

DO NOW

Find the second derivative of: $y^2 = x^2 + 2x$

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3.5 Implicit Differentiation- Day 3

Logarithmic Differentiation:

- ** Take \ln of both sides
- ** Use to differentiate sums and differences instead of products and quotients.
- ** Put in terms of x only by substituting the original $y = \text{value}$ into the derivative if possible.

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Use logarithmic differentiation to find the derivative.

$$\begin{aligned}
 1. \quad y &= x^{x-1} \\
 \ln y &= \ln x^{x-1} \\
 \ln y &= (x-1) \ln x \\
 \frac{1}{y} \cdot y' &= (x-1) \cdot \frac{1}{x} + \ln x (1) \\
 y' &= y \left[\frac{x-1}{x} + \ln x \right] \\
 y' &= x^{x-1} \left[\frac{x-1}{x} + \ln x \right] \\
 y' &= x^{x-1} \left(\frac{1}{x} \right) [x-1 + x \ln x] \\
 y' &= \frac{x^{x-1}}{x} (x-1 + x \ln x) \\
 y' &= x^{x-2} (x-1 + x \ln x)
 \end{aligned}$$

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$$\begin{aligned}
 2. \quad y &= \sqrt[3]{\frac{x^2+1}{x^2-1}} \\
 \ln y &= \ln \sqrt[3]{\frac{x^2+1}{x^2-1}} \\
 \ln y &= \frac{1}{3} \ln \left(\frac{x^2+1}{x^2-1} \right) \\
 \ln y &= \frac{1}{3} (\ln(x^2+1) - \ln(x^2-1)) \\
 \frac{y'}{y} &= \frac{1}{3} \left(\frac{2x}{x^2+1} - \frac{2x}{x^2-1} \right) \\
 \frac{y'}{y} &= \frac{1}{3} \left(\frac{2x(x^2-1) - 2x(x^2+1)}{(x^2+1)(x^2-1)} \right) \\
 \frac{y'}{y} &= \frac{1}{3} \left(\frac{2x^3 - 2x - 2x^3 - 2x}{x^4 - 1} \right) \\
 y' &= \frac{1}{3} (y) \left(\frac{-4x}{x^4 - 1} \right)
 \end{aligned}$$

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HOMEWORK

pg 173; 56, 63, 65 - 67, 71, 73

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